Problem Set 2 Solution

1. $$F(E) = \frac{1}{1 + e^{(E - E_f / kT)}}$$

(a) $$F(E) = \frac{1}{1 + e^{(E_c + kT - E_c / kT)}} = \frac{1}{1 + e^{(1)}} = 0.269$$

state occupied

(b) $$1 - F(E) = 1 - \frac{1}{1 + e^{(E_V - kT - E_V / kT)}} = 1 - \frac{1}{1 + e^{(-1)}} = 0.269$$

state empty

---

2. The Boltzmann approximation : $F(E) \cong e^{-(E - E_f)/kT}$

$$\% \text{ error} = \frac{\text{approximated} - \text{actual}}{\text{actual}}$$

* Remember $e^{-x} \equiv \frac{1}{e^x}$

$$= \frac{\frac{1}{e^{(E-E_f)/kT}} - \frac{1}{1 + e^{(E-E_f)/kT}}}{\frac{1}{1 + e^{(E-E_f)/kT}}} = \frac{1 + e^{(E-E_f)/kT}}{e^{(E-E_f)/kT}} - 1$$

$$= e^{-(E-E_f)/kT} + 1 - 1$$

$$0.01 = e^{-(E-E_f)/kT}$$

$$\ln(0.01) = -\frac{(E - E_f)}{kT}$$

$$E = E_f - kT \ln(0.01)$$

$$E = E_f + 4.6kT$$

(b) $$F(E) = \frac{1}{1 + e^{\left(\frac{E - E_f}{kT}\right)}} = \frac{1}{1 + e^{(4.6)}} = 9.95 \times 10^{-3}$$

(3) $E_f = 6.25\,eV \qquad T = 300K$

(a) $F(E) = \dfrac{1}{1 + e^{\left(\frac{6.50 - 6.25}{0.0259}\right)}}$ $\qquad E = 6.50\,eV$

$= 6.43 \times 10^{-5}$

(b) $F(E) = \dfrac{1}{1 + e^{\left(\frac{(6.50 - 6.25) \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 950}\right)}}$ $\qquad T = 950$

convert eV to J!

$= 0.0452$

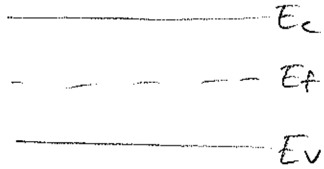
(c) $1 - 0.01 = \dfrac{1}{1 + e^{(-0.3 \times 1.6 \times 10^{-19}/kT)}}$ $\qquad E - E_f = -0.3\,eV$

$\dfrac{1}{0.99} - 1 = e^{-0.3/kT} \qquad \ln\left(\dfrac{1}{99}\right) = \dfrac{-0.3 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}\ T}$

$T = \dfrac{-0.3 \times 1.6 \times 10^{-19}}{\ln\left(\frac{1}{99}\right) \times 1.38 \times 10^{-23}} \cong 757\,K$

---

(4) (a) E_g for Si $= 1.12\,eV$



$F(E) = \dfrac{1}{1 + e^{(E - E_f/kT)}}$

$E_c - E_f = \dfrac{E_g}{2}$ (for this problem)

$F(E) = \dfrac{1}{1 + e^{(0.56/0.0259)}} = 4.07 \times 10^{-10}$

for Ge: $F(E) = \dfrac{1}{1 + e^{(0.33/0.0259)}} = 2.93 \times 10^{-6}$

for GaAs: $F(E) = \dfrac{1}{1 + e^{(0.71/0.0259)}} = 1.24 \times 10^{-12}$

(b) The same as part (a). The probability of a state at at $E_1 = E_f + \Delta E$ being occupied is the same as the probability of a state at $E_2 = E_f - \Delta E$ being empty. 3

5 $$F(E) = \frac{1}{1 + e^{(E-E_f)/kT}}$$

$$1 + e^{(E-E_f)/kT} = \frac{1}{F(E)}$$

$$\frac{E - E_f}{kT} = \ln\left(\frac{1}{F(E)} - 1\right)$$

$$T = \frac{0.55 \times 1.6 \times 10^{-19}}{\ln\left(\frac{1}{10^{-6}} - 1\right) \times 1.38 \times 10^{-23}} \cong 462K$$